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THERMAL RESISTANCE OF A SYSTEM OF PARALLELEPIPEDS

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UDC 536.248.1

A method is proposed for calculating the thermal resistance of a system of parallelepipeds with a local source which is encountered in the analysis of the thermal conditions in hybrid integral microcircuits.

Many applied problems, particularly problems associated with the analysis of thermal conditions in microelectronic devices, reduce to a thermal model which is a "pyramid" of n unlike parallelepipeds of different sizes (Fig. 1a). In the actual construction, the parallelepipeds forming the pyramid are the backing, "chips," adhesive layer, switching plate, base, etc. [1]. There are rectangular, flat energy sources on the upper surface of the first parallelepiped. Thermal contact between adjacent surfaces is assumed ideal. Heat dissipation from the lower surface of the n-th parallelepiped obeys Newton's law and is characterized by a heat-exchange coefficient α; there is no heat exchange at the lower surfaces.

The exact mathematical description of the temperature field in such a system is rather complex and hardly can be used for practical purposes. Calculation of the thermal resistance from the source to the environment is usually based on the construction of an equivalent circuit representing a chain of series-connected thermal resistances [1-4]. It is further assumed that the interfaces are isothermal.

We analyzed the possibility of such an approach for the following problem: a bounded cylinder with a local energy source on one end and boundary conditions of the first and third kind on the opposite end. The thermal resistance from the source to the environment was calculated in the two cases. A comparison of the results showed that the values of the thermal resistance can differ by almost a factor of two for given values of the Biot number and given ratios of cylinder and source sizes. Therefore, determination of the thermal resistance of this system must be carried out with consideration of heat-transfer conditions at the heat-releasing surfaces of each body. A method is proposed below for which sequential application provides an accuracy sufficient for practical purposes without significant complication of the computational process.

Method for Determination of Thermal Resistance

We shall show that for this system of bodies, the problem of determining the total thermal resistance from energy source to environment can be reduced to a problem of determining the thermal resistance of the first parallelepiped, the heat-transfer conditions at the lower boundary of which are characterized by an equivalent coefficient α_1 that includes the effect of all the other parallelepipeds (Fig. 1c).

To determine the value of α_1, we consider successively the temperature field of each i-th parallelepiped, for which the thermal model can be represented in the following manner: on the upper face of the parallelepiped, there is a flat energy source with an intensity p and area S_{i-1}; on the lower surface, heat transfer is characterized by a heat-transfer coefficient α_i, which takes into account the effect of all the remaining $(n-i)$

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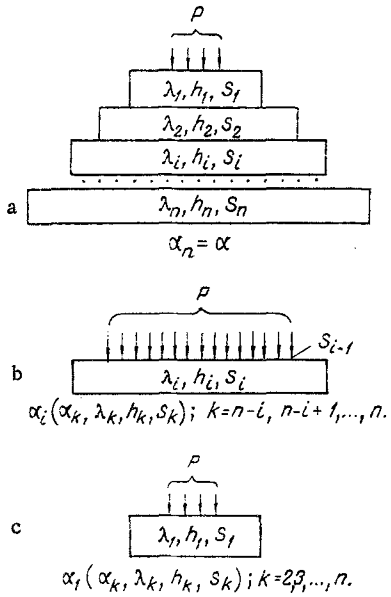

Fig. 1. Thermal models (a, b, c) for calculating the thermal resistance of hybrid integral microcircuits.

parallelepipeds with the other surfaces thermally insulated (Fig. 1b).

For the lowest parallelepiped (i = n), the quantity α_n is known and characterizes the actual conditions of heat exchange with the environment, $\alpha_n = \alpha$. We define the thermal resistance of the n-th parallelepiped from its contact surface to the environment in the following manner:

$$R_n = \frac{h_n}{\lambda_n S_{n-1}} \gamma_n \, (\alpha_n, \, h_n, \, S_n, \, S_{n-1}). \tag{1}$$

Here γ_n is a coefficient which takes into account spreading of the thermal flux over the thickness of the parallelepiped.

To determine α_{n-1}, the equivalent coefficient of heat exchange for the (n − 1)-th parallelepiped, we represent the quantity R_n in the form

$$R_n = \frac{1}{\alpha_{n-1} S_{n-1}}. \tag{2}$$

We then have from Eqs. (1) and (2)

$$\alpha_{n-1} = \frac{\lambda_n}{h_n \gamma_n (\alpha_n, \, h_n, \, S_n, \, S_{n-1})}. \tag{3}$$

Now considering the (n − 1)-th parallelepiped, we express its thermal resistance similarly through the corresponding geometric parameters, coefficient of thermal conductivity λ_{n-1}, and coefficient of heat exchange α_{n-1}. Repeating this operation successively for each i-th parallelepiped, we obtain

$$\alpha_i = \frac{\lambda_{i+1}}{h_{i+1} \gamma_{i+1} (\alpha_{i+1}, \, h_{i+1}, \, S_{i+1}, \, S_i)}, \tag{4}$$

$$R_i = \frac{h_i}{\lambda_i S_{i-1}} \gamma_i (\alpha_i, \, h_i, \, S_i, \, S_{i-1}). \tag{5}$$

In the final analysis, we obtain the equivalent heat-exchange coefficient α_1 at the boundary between the first and second parallelepipeds (Fig. 1c). Then the thermal resistance from the source on the upper surface of the first parallelepiped to the environment has the form

$$R_1 = \frac{h_1}{\lambda_1 S_s} \gamma_1 (\alpha_1, \, h_1, \, S_1, \, S_s). \tag{6}$$

Analytic expressions for the determination of the quantities γ_i can be obtained by both exact [5] and approximate [6, 7] methods. Furthermore, it is usually assumed that the thermal flux density in the region occupied by the source is uniformly distributed, i.e., q = const.

In this problem, the assumption q = const can be considered valid only for the top parallelepiped, on the surface of which the actual sources are located. For the remaining parallelepipeds, the thermal flux density

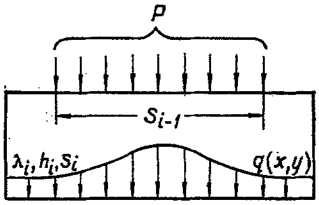

Fig. 2

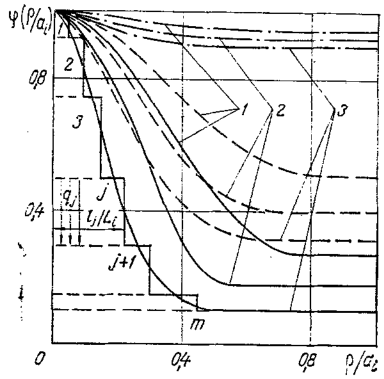

Fig. 3

Fig. 2. Thermal flux density distribution at contact surfaces.

Fig. 3. Form of the function $\varphi(\rho/a_i)$ for various parameter values: 1) $l_i/L_i = 0.5$; 2) 0.3; 3) 0.1; solid curve, $Bi_i = 0.2$, $h_i/L_i = 0.05$; dashed curve, $Bi_i = 0.2$, $h_i/L_i = 0.1$; dashed-dot curve, $Bi_i = 0.01$, $h_i/L_i = 0.1$.

distribution at the contact surface is nonuniform, $q(x, y)$, and depends on heat-exchange conditions and the ratio of parallelepiped sizes. In order to use the analytic expressions for γ_i obtained for $q = \text{const}$, therefore, it is necessary to check the validity of this condition for the calculation of the thermal resistance of each i-th parallelepiped in this system. This check must include the following: determination of the nature of the distribution $q(x, y)$ as a function of the Biot number and of the ratio of parallelepiped sizes; a quantitative evaluation of the nonuniformity of the thermal flux density distribution; and an evaluation of the error in the determination of the thermal resistance of a parallelepiped as a function of the nonuniformity index selected.

Evaluation of Nonuniformity of Thermal Flux Density

We consider the thermal model of the i-th parallelepiped (Fig. 1b). We first assume that the density of the energy source located on the upper surface is constant and equal to q_0. We investigate the nature of the thermal flux density distribution $q(x, y)$ at the lower surface (Fig. 2) on the basis of the Biot number Bi_i and the ratio of source and parallelepiped sizes. We represent the dependence $q(x, y)$ in the following form:

$$q(x, y) = q_0 \mu(x, y). \tag{7}$$

In accordance with the principle of local effect, we make an approximate substitution for the functions of two variables $q(x, y)$ and $\mu(x, y)$ with functions of a single variable,

$$q(x, y) \approx q(\rho); \ \mu(x, y) \approx \mu(\rho); \ \rho = \sqrt{x^2 + y^2}.$$

Considering, in particular, a square source of length l_i on a side located in the center of the upper face of a square plate of length L_i on a side and thickness h_i, we represent the function $\mu(\rho)$ in the dimensionless form

$$\mu\left(\frac{\rho}{a_i}\right) = \mu(0)\varphi\left(\frac{\rho}{a_i}\right), \ L_i^2 = \pi a_i^2. \tag{8}$$

Here $\varphi(\rho/a_i)$ is a normalized function which determines the nature of the thermal flux density distribution on the lower surface; $\mu(0)$ is the value of μ at the source center $\rho = 0$. The function $\varphi(\rho/a_i)$ is shown in Fig. 3. The curves in this figure were obtained by exact solution of the appropriate problem of thermal conductivity for various individual parameter ratios h_i/L_i, l_i/L_i, and Bi_i.

The nonuniformity in thermal flux distribution can be quantitatively evaluated by various methods. The most general results are obtained if the following relation is selected as the nonuniformity criterion:

$$\eta = \frac{q_s}{q_a}. \tag{9}$$

Here q_s is the mean surface thermal flux density

$$q_s = \frac{1}{\pi a_i^2} \int_0^{a_i} q(\rho) 2\pi\rho d\rho,$$

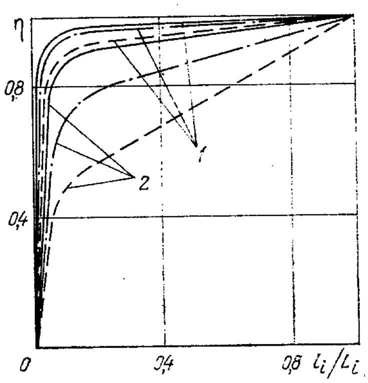

Fig. 4

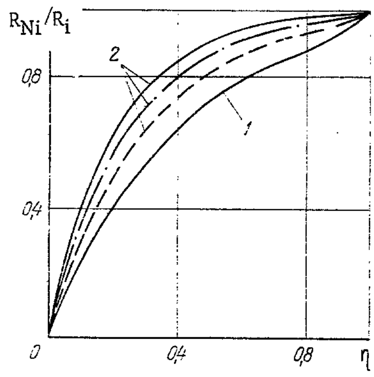

Fig. 5

Fig. 4. Dependence of nonuniformity criterion η on various factors: 1) $Bi_i = 0.01$; 2) 0.2; solid curve, $h_i/L_i = 0.2$; dashed-dot curve, $h_i/L_i = 0.1$; dashed curve, $h_i/L_i = 0.05$.

Fig. 5. Dependence of thermal resistance of a parallelepiped on nonuniformity of thermal flux in the source region: 1) $Bi_i = 0$–0.1, $h_i/L_i \geq 0.05$; 2) $Bi_i = 10$; dashed curve, $h_i/L_i = 0.2$; dashed-dot curve, $h_i/L_i = 0.1$; solid curve, $h_i/L_i = 0.05$.

and q_a is the thermal flux density averaged over the radius

$$q_a = \frac{1}{a_i}\int_0^{a_i} q(\rho)\,d\rho.$$

Keeping Eqs. (7) and (8) in mind, we obtain

$$\eta = 2\frac{\int_0^1 \varphi\left(\frac{\rho}{a_i}\right)\frac{\rho}{a_i}\,d\left(\frac{\rho}{a_i}\right)}{\int_0^1 \varphi\left(\frac{\rho}{a_i}\right)d\left(\frac{\rho}{a_i}\right)}. \tag{10}$$

Figure 4 shows values of the criterion η obtained from Eq. (10) by numerical integration of the functions $\varphi(\rho/a_i)$ (Fig. 3) and $\varphi(\rho/a_i)(\rho/a_i)$ for the following parameter ranges: $l_i/L_i = 0$–1; $h_i/L_i = 0$–0.2; $Bi_i = 0$–0.2.

Thermal Resistance as a Function of the Parameter η

We now consider the effect of nonuniformity in the thermal flux distribution from the source on the thermal resistance of the parallelepiped. We assume there is a source on the upper face of the i-th parallelepiped with a nonuniform thermal flux density which is characterized by the relation shown in Fig. 3. We point out that this relation is a step function consisting of m steps as shown in Fig. 3. Then the energy source can be represented as a sum of m sources of different sizes l_i/L_i and a uniform density q_j, the value of which is

$$q_j = q_0\mu(0)\left[\varphi_j\left(\frac{\rho}{a_i}\right) - \varphi_{j-1}\left(\frac{\rho}{a_i}\right)\right]. \tag{11}$$

The mean surface heating of the region S_j occupied by each of the j sources is ϑ_{js}. The actual energy source occupies the area S_m corresponding to j = m. Each of the j sources increases the mean heating of the actual area S_m by $\vartheta_{js} = S_j/S_m$. Then the mean surface heating by the source, ϑ_s, on the basis of the superposition principle is

$$\vartheta_s = \sum_{j=1}^{m}\vartheta_{js}\frac{S_j}{S_m}. \tag{12}$$

By definition, the value of ϑ_{js} is

$$\vartheta_{js} = q_j S_j R_j, \tag{13}$$

TABLE 1. Input Data for the Calculation of the Thermal Resistance of a System of Cylinders

Cylinder number	1	2	3	4	5
Coefficient of thermal conductivity, λ_i, W/m·°K	2	1	2	1	20
Height h_i, m	$0{,}2\cdot10^{-3}$	$0{,}3\cdot10^{-3}$	$0{,}4\cdot10^{-3}$	$0{,}5\cdot10^{-3}$	$0{,}5\cdot10^{-3}$
Diameter $2a_i$, m	$4\cdot10^{-3}$	$6\cdot10^{-3}$	$8\cdot10^{-3}$	$10\cdot10^{-3}$	$10\cdot10^{-3}$

where the thermal resistance R_j on the basis of Eq. (5) has the form

$$R_j = \frac{h_i}{\lambda_i S_j}\gamma_{ij}. \tag{14}$$

Then the thermal resistance of a parallelepiped for nonuniform thermal flux distribution is

$$R_{Ni} = \frac{\vartheta_c}{P} = \frac{t_s - t_c}{P}. \tag{15}$$

We compare the value of the thermal resistance R_{Ni} for the i-th parallelepiped as determined from Eq. (15) with the value of the thermal resistance R_i for the same parallelepiped with a uniform thermal flux distribution as determined from Eq. (5). Keeping Eqs. (11)-(15) in mind and carrying out the necessary algebraic transformations, we finally obtain

$$\frac{R_{Ni}}{R_i} = \frac{\sum_{j=1}^{m}\left[\varphi_j\left(\frac{\rho}{a_i}\right) - \varphi_{j+1}\left(\frac{\rho}{a_i}\right)\right]\frac{S_j\gamma_{ij}}{S_m\gamma_{im}}}{\sum_{j=1}^{m}\left[\varphi_j\left(\frac{\rho}{a_i}\right) - \varphi_{j+1}\left(\frac{\rho}{a_i}\right)\right]\frac{S_j}{S_m}}. \tag{16}$$

The quantity R_{Ni}/R_i characterizes the effect of thermal flux nonuniformity on the thermal resistance of a parallelepiped. Figure 5 shows the dependence of the ratio R_{Ni}/R_i on the value of the criterion η for various values of the other parameters h_i/L_i, l_i/L_i, and Bi_i.

Analysis of the results makes possible the following conclusions.

1. The thermal resistance depends only on the criterion η in the region $h_i/L_i > 0.05$, $Bi_i < 0.1$; the thermal resistance depends on η and on Bi_i in the region $h_i/Li_i > 0.2$, $Bi_i > 0.1$; the thermal resistance depends on Bi_i, η, and h_i/L_i for very small relative thickness of a parallelepiped ($h_i/L_i < 0.05$). Consequently, one can pick out a considerable range of parameters in which the thermal resistance depends only on the criteria η and Bi_i, which confirm the advisability of selecting the criterion η for evaluation of thermal flux nonuniformity in the source region.

2. The effect of thermal flux nonuniformity of contact surfaces can be neglected for $\eta > 0.6$. The error in the determination of the thermal resistance of an individual parallelepiped does not exceed 25%, which is acceptable in many cases.

From Figs. 4 and 5, one can estimate the value of the correction R_{Ni}/R_i without resorting to a calculation of the thermal flux distribution at contact surfaces and take it into account in the calculation of the total thermal resistance of a system of bodies.

Evaluation of Accuracy of the Method

It is practical to compare calculated results for the thermal resistance of a system of bodies obtained by the proposed method with calculated results obtained from the method of thermal resistances connected in series and also with similar results obtained by a numerical method. The capabilities of an average type of computer do not permit numerical calculation of the three-dimensional temperature field for such complex systems. Such a comparison was therefore performed for a system of bodies of cylindrical shape having thermophysical properties and dimensions given in Table 1. The ratios of sizes and of coefficients of thermal conductivity were selected to be similar to those encountered in actual microcircuits. The radius of the energy source on the upper surface of the first cylinder was $1 \cdot 10^{-3}$ m and the specific power was $q_0 = 2 \cdot 10^5$ W/m^2.

The calculation determined the heat of the center of the source with respect to the lower surface of the last cylinder, for which the temperature field can be assumed uniform ($\lambda_5 = 20$ W/m·°K). A comparison of the values for the heating gives the following results: numerical method, 108.9°; proposed method including thermal flux nonuniformity at contact surfaces, 109°; proposed method without inclusion of nonuniformity, 112°; method of series-connected thermal resistances, 49°.

A comparison of the results shows that calculation by the proposed method can be made in some cases without consideration of thermal flux nonuniformity at contact surfaces, which leads to smaller errors than the assumption of isothermicity at those surfaces. In addition, an evaluation of the accuracy of the method was made on the basis of a comparison of the results of an experimental determination and of a calculation by the proposed method of the thermal resistance of actual microcircuits. The divergence of the results did not exceed the spread in the value of the thermal resistance of a microcircuit caused by instability in technical procedures.

NOTATION

P, power of energy source; q, flux density; λ_i, coefficient of thermal conductivity of the i-th parallelepiped; S_i, S_{i-1}, areas of the i-th and (i − 1)-th parallelepipeds; h_i, height of the i-th parallelepiped; $Bi_i = \alpha_i h_i / \lambda_i$, Biot number of the i-th parallelepiped; t_c, ambient temperature; $\vartheta = t - t_c$, heating.

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THEORY OF NEW KINETIC METHODS OF MEASURING THE MASS-TRANSFER PROPERTIES OF DISPERSED SOLIDS

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UDC 66.047.35

The solution of the generalized mass-transfer equations with variable coefficients is used as a basis for developing the theory of various new methods of determining the transfer characteristics of dispersed solids with finite moisture contents.

Methods of experimentally determining the mass-transfer characteristics of highly dispersed, moisture-containing solids employed at the present time are subject to a number of fundamental shortcomings which greatly reduce the accuracy and reliability of the results obtained. Practically all the methods of measuring mass-transfer characteristics are analogous to the corresponding heat-transfer methods. At the same time, mass transfer differs very considerably from heat transfer, despite the identical nature of the transfer equations. Thus, the use of any particular solution of the heat-conduction equation as a basis for the development of methods of determining mass-transfer coefficients is frequently unreliable, even though the corresponding thermophysical procedure has been thoroughly vindicated.

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